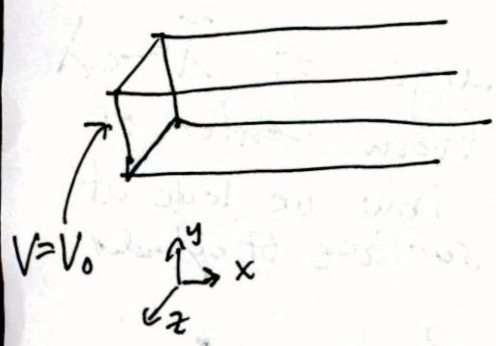
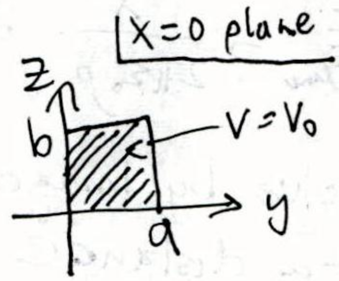
Phys 100A Discussion Notes - Week 7

Ex 3.5 Griffiths'





$$\nabla^2 V = 0 \Rightarrow \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = 0$$

Seperable solution: $\frac{1}{X}\frac{d^2X}{dx^2} + \frac{1}{Y}\frac{d^2Y}{dy^2} + \frac{1}{Z}\frac{d^2Z}{dz^2} = 0$

$$\underset{C_1}{\overset{\parallel}{}} + \underset{C_2}{\overset{\parallel}{}} + \underset{C_3}{\overset{\parallel}{}} = 0$$

Physically, $C_1 > 0$, $C_2, C_3 < 0$. Why?

Define $C_2 = -k^2$ $\quad C_3 = -\ell^2$ then

$$\frac{d^2X}{dx^2} = (k^2+\ell^2)X \qquad \frac{d^2Y}{dy^2} = -k^2 Y \qquad \frac{d^2Z}{dz^2} = -\ell^2 Z$$

From the B.C. and limiting behavior

$$V(x,y,z) = \sum_{n,m=1}^{\infty} C_{nm}\, e^{-\pi\sqrt{(n/a)^2+(m/b)^2}\,x} \sin\frac{n\pi y}{a} \sin\frac{m\pi z}{b}$$

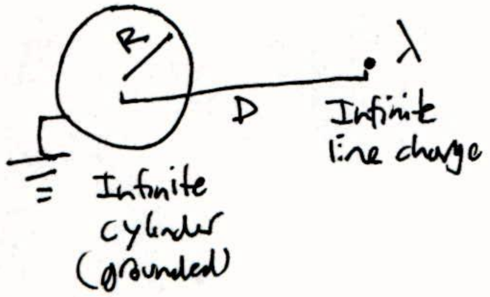
$$V_0 = V_0(y,z) = \sum_{n,m=1}^{\infty} C_{nm} \sin\frac{n\pi y}{a} \sin\frac{m\pi z}{b}$$

can depend on y, z

$$\Rightarrow C_{n,m} = \frac{4}{ab}\int_0^a dy \int_0^b dz\, V_0(y,z) \sin\left(\frac{n\pi y}{a}\right)\sin\left(\frac{m\pi z}{b}\right)$$

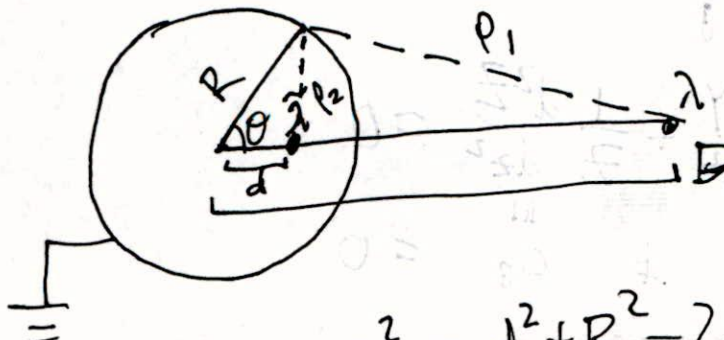
Constant case?

Method of images



$$\vec{E}_{line} = \frac{\lambda}{2\pi\varepsilon_0 \rho}\hat{\rho} \Rightarrow V = \frac{-\lambda \ln\rho}{2\pi\varepsilon_0}$$

Solve by image charge of $\tilde{\lambda} = -c\lambda$ at a distance d from center of cylinder. Suppose for now we look at the potential on the surface of cylinder.



$$V = \frac{-\lambda}{2\pi\varepsilon_0}\left(\ln\frac{\rho_1}{\rho_2^c}\right) + V_0$$

Using geometry we have ...

$c^2 = a^2 + b^2 - 2ab\cos C$

$$\rho_2^2 = d^2 + R^2 - 2Rd\cos\theta$$
$$\rho_1^2 = D^2 + R^2 - 2RD\cos\theta$$

Now choose $d = b\frac{R^2}{D}$, $b < 1$

$$V = \frac{-\lambda}{4\pi\varepsilon_0}\ln\left(\frac{D^2+R^2-2RD\cos\theta}{\left(b^2\frac{R^4}{D^2}+R^2-2bR^3\frac{\cos\theta}{D}\right)^c}\right) + V_0$$

$$= \frac{-\lambda}{4\pi\varepsilon_0}\ln\left(\left(\frac{D^2}{R^2}\right)^c \frac{D^2+R^2-2RD\cos\theta}{\left(b^2R^2+D^2-2bRD\cos\theta\right)^c}\right) + V_0$$

$c = 1$, $b = 1$, $\boxed{V_0 = \frac{\lambda}{4\pi\varepsilon_0}\ln\frac{D^2}{R^2}}$

$= 0 \quad \forall\theta \Rightarrow$

Check: $\vec{E} = -\nabla V = \frac{1}{R}\frac{\partial V}{\partial\theta}\hat{\theta} + \frac{\partial V}{\partial\rho}\hat{\rho}$ is $\perp$ to cond. surface