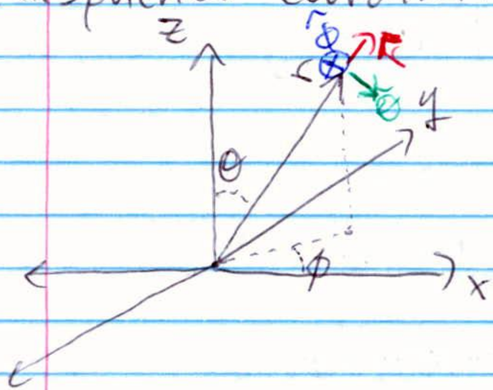
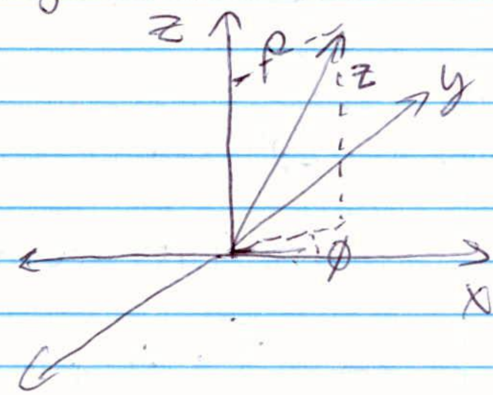
Phys 100A
Week One – Discussion Notes

1 (a) Spherical Coordinates — Cylindrical Coordinates





What are the unit vectors in each coordinate systems?
- → point in the direction of max increase of coordinate
- → have length one

Cartesian case: $\rightarrow \hat{x}$ $\quad \uparrow \hat{y}$ $\quad \odot z$

Position independent! Not always so..

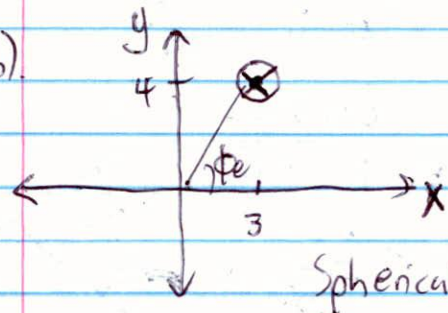
$\hat{r} = \frac{\vec{r}}{r}$ $\qquad \vec{r} = \begin{pmatrix} r\sin\theta\cos\phi \\ r\sin\theta\sin\phi \\ r\cos\theta \end{pmatrix}$

Thus we see $\quad \hat{r} = \hat{r}(\theta,\phi)$

The same holds for θ, ϕ $\qquad \hat{\phi} = \hat{\phi}(\phi) \quad \hat{\theta} = \hat{\theta}(\theta)$

So $\quad (-1,0,0) \longrightarrow \hat{r}\left(\frac{\pi}{2}, \pi\right)$

(b)



cylindrical: Electron position: $5\hat{\rho}(\phi_e)$

Electron velocity $-\hat{z}$

Spherical: Electron position $5\hat{r}\left(\frac{\pi}{2}, \phi_e\right)$

Electron velocity: $\hat{\theta}$

(1c) $f(x,y,z) = xy$

Cartesian: $\vec{\nabla} f = \begin{pmatrix} y \\ x \\ 0 \end{pmatrix} \underset{@}{=} \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$

Spherical:

- First rewrite in spherical coordinates

$f(r,\theta,\phi) = r^2 \sin^2\theta \cos\phi \sin\phi$

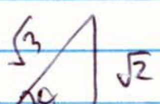
Recall

$$\vec{\nabla} f = \frac{\partial f}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial f}{\partial \theta}\hat{\theta} + \frac{1}{r\sin\theta}\frac{\partial f}{\partial \phi}\hat{\phi}$$

$$\rightarrow \vec{\nabla} f = 2r\sin^2\theta\cos\phi\sin\phi\,\hat{r}$$
$$+ 2r\sin\theta\cos\theta\cos\phi\sin\phi\,\hat{\theta}$$
$$+ r\sin\theta\cos 2\phi\,\hat{\phi}$$

We now want to evaluate this at $(1,1,1)$

$r = \sqrt{3}$, $\arctan\left(\frac{\sqrt{2}}{1}\right) = \theta$, $\phi = \arctan\left(\frac{1}{1}\right) = \frac{\pi}{4}$



$\sin\theta = \sqrt{\frac{2}{3}}$

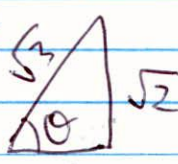
$\cos\theta = \frac{1}{\sqrt{3}}$

$\sin\phi = \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}$

$\cos\phi = \frac{1}{\sqrt{2}}$

~~Plugging in we get~~

~~[illegible]~~



$\sin\theta = \sqrt{\frac{2}{3}}$

$\cos\theta = \frac{1}{\sqrt{3}}$

$\cos 2\theta = -1/3$

$\sin\phi = \cos\phi = 1/\sqrt{2}$

$$\vec{\nabla} f = \sqrt{2}\left(2\frac{1}{\sqrt{6}}\hat{r}(\theta,\phi) + 2\frac{1}{2\sqrt{3}}\hat{\theta}(\phi) + \cancel{\frac{1}{3}\hat{\phi}(\phi)}\right)$$

Phys 100A - Week One Discussion

2. (a). $(1+x)^n \approx 1+nx$ if $|x| \ll 1$

(expanding around $x=0$)!

Recall

$$f(x) = f(x_0) + (x-x_0)f'(x_0) + \frac{1}{2}(x-x_0)^2 f''(x_0) \ldots$$

$x_0 = 0$ $\quad f(0) = 1$

$$f'(0) = n(1+x)^{n-1} = n$$

$$f''(0) = n(n-1)(1+x)^{n-2} = n(n-1)$$

So $(1+x)^n \sim 1+nx$

Then

$$\sqrt{1+x} = (1+x)^{1/2} = 1 + \frac{1}{2}x + O(x^2)$$

(b). $\sqrt{101} = \sqrt{100+1} = 10\sqrt{1+\frac{1}{100}} \approx 10\left(1+\frac{1}{200}\right)$

(c). $f(z) = \dfrac{kqz}{(z^2+R^2)^{3/2}} = \dfrac{kq}{z^2}\left(1+\left(\dfrac{R^2}{z^2}\right)\right)^{-3/2}$

So

$$f(z) \sim \frac{kq}{z^2}\left(1 - \frac{3}{2}\left(\frac{R^2}{z^2}\right)\right)$$

(d). $PE = \dfrac{-GMm}{R+h} = -\dfrac{GMm}{R}\dfrac{1}{1+h/R} \approx -\dfrac{GMm}{R}(1-h/R)$