Many-body localization and Spontaneous Symmetry Breaking in the Massless Schwinger Model

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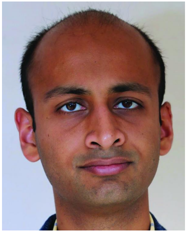
December 14, 2018

Outline

Some history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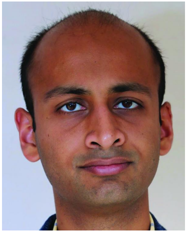


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 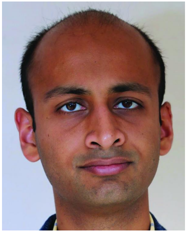 

- Began as a senior thesis project during Undergrad at Princeton.
- Advised by Shivaji Sondhi (Princeton) and Mari Carmen Banuls (MPQ).
- Motivated by arguments published by Nandkishore and Sondhi (PRX 2017) [1].

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- Goal: investigate lattice formulation (where there is no analytic solution via bosonization), where commensurate effects are also present [7].

How to regularize?

The continuum Hamiltonian formulation in the temporal gauge [8]

$$H = \int dx \{ \bar{\Psi} [i\gamma^1 \partial_1 + e\gamma^1 A_1 + \mu(x)] \Psi + \frac{1}{2} E^2 \} \tag{3}$$

$$\rightarrow H = \frac{-i}{2a} \sum_n (\phi_n^\dagger e^{i\theta_n} \phi_{n+1} - h.c.) + \frac{ag^2}{2} \sum_n L_n^2 \tag{4}$$

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- **Kogut-Susskind staggered (SB) potential (CR)** $\iff$ (2CCP)

$$L_n - L_{n-1} = \phi_n^\dagger \phi_n - \frac{1}{2}[1 - (-1)^n] \qquad \langle \phi_n^\dagger \phi_n \rangle = n \mod 2 \tag{5}$$

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- **Translationally invariant potential (SR)** $\iff$ (1CCP)

$$L_n - L_{n-1} = \phi_n^\dagger \phi_n - 1/2 \qquad \langle \phi_n^\dagger \phi_n \rangle = 0 \tag{6}$$

- Jordan-Wigner-ize $\phi_n \rightarrow \prod_{m<n}(i\sigma_m^z)\sigma_n^-$, $\phi_n^\dagger \phi_n \rightarrow \frac{1}{2}(\sigma^z + 1)$

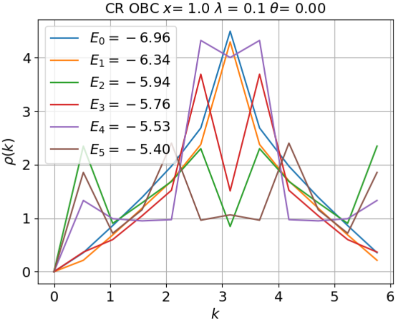
Lattice Hamiltonian

	Open boundary conditions	Periodic boundary conditions
H_{hop}	$\sum_{n=0}^{N-2}(\sigma_n^+\sigma_{n+1}^- + \sigma_n^-\sigma_{n+1}^+)$	$\sum_{n=0}^{N-1}(\sigma_n^+\sigma_{n+1}^- + \sigma_n^-\sigma_{n+1}^+)$
H_{int}	$\sum_{n=0}^{N-2}(E_n)^2$	$-\sum_{j<i}\rho(i)\rho(j)\sin\left[\frac{\pi}{N}(i-j)\right]$
Conventional	$E_n = \frac{1}{4}\sum_{k=0}^{N-1}\operatorname{sgn}\left(n+\frac{1}{2}-k\right)\sigma_k^z + \frac{1}{4}[(-1)^n+1]$	$\rho(n) = \frac{1}{2}\left[\sigma_n^z + (-1)^n\right]$
Symmetric	$E_n = \frac{1}{4}\sum_{k=0}^{N-1}\operatorname{sgn}\left(n+\frac{1}{2}-k\right)\sigma_k^z$	$\rho(n) = \frac{1}{2}\sigma_n^z$

$$H = xH_{hop} + \lambda H_{int} + \frac{1}{2}\sum_{n=0}^{N-1} V(n)\sigma_n^z \qquad V(n) \in [-\theta, \theta] \tag{7}$$

$$\left(\sum_n \sigma_n^z\right)|j\rangle = 0 \tag{8}$$

Fourier-transformed density-density correlations ($N = 12$)



$$\rho_n(k) = \sum_{r=0}^{N-1} \langle \sigma_0^z \sigma_r^z \rangle_n e^{ikr} \quad (9)$$

States calculated by E.D. for $N = 12$, Lanczos' algorithm for higher N.

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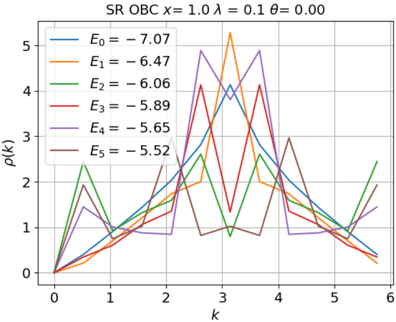

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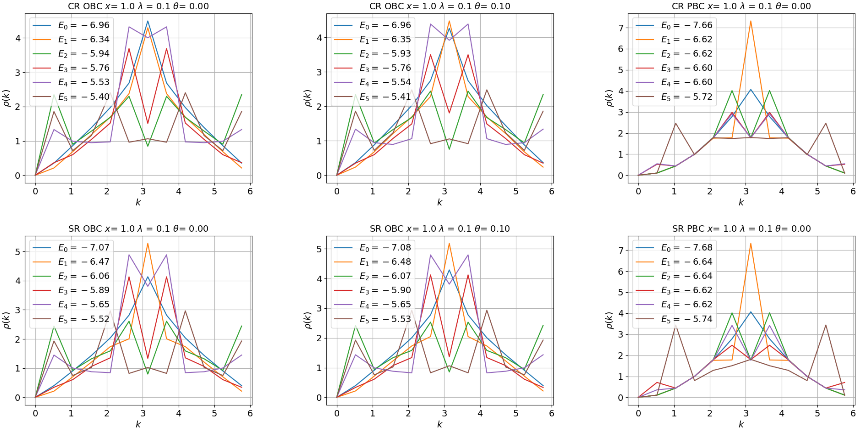

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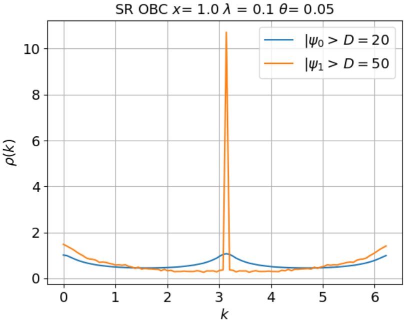

$$\rho_n(k) = \sum_{r=0}^{N-1} \langle \sigma_0^z \sigma_r^z \rangle_n e^{ikr} \tag{11}$$

States calculated using iterative ground state search in DMRG. Excited states calculated via penalty, as described in [8].

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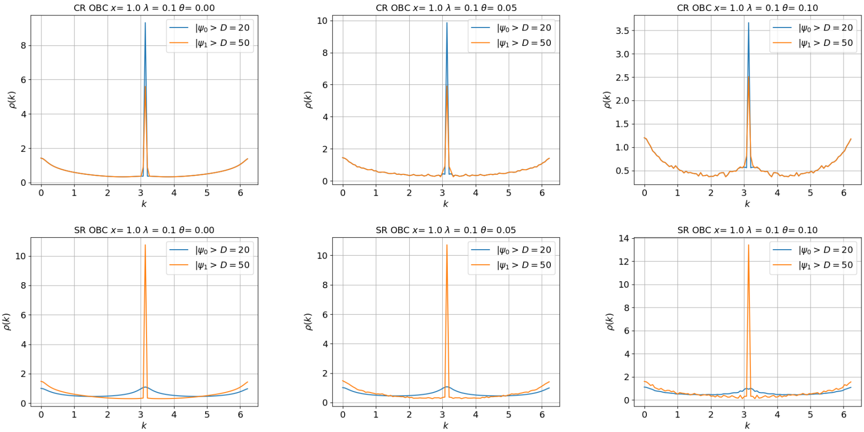

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  The classical 1CCP develops long-range crystalline order at any temperature as well.

What is the level statistics ratio?

$$\tilde{r}_i = \frac{\min(E_{i+2} - E_{i+1}, E_{i+1} - E_i)}{\max(E_{i+2} - E_{i+1}, E_{i+1} - E_i)} \tag{12}$$

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  Spectra of time reversal-invariant systems whose classical analogues are K systems show the same fluctuation properties as predicted by GOE.

Level statistics

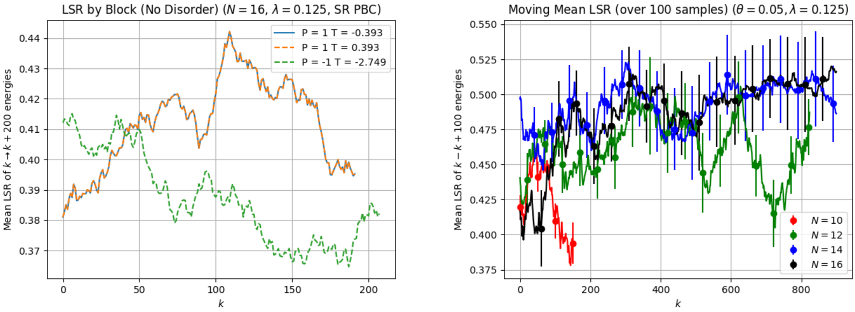
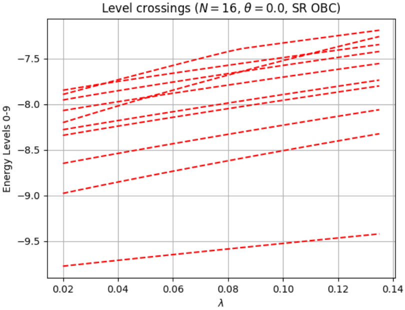
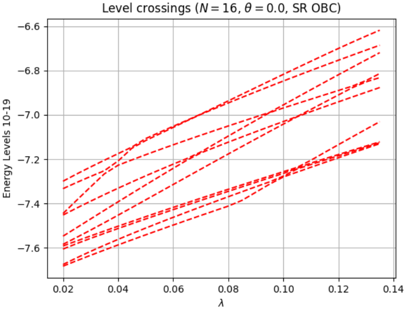


Figure: Window size is 100 energies. Note $\theta = 0$ statistics are far from ergodic. Also, note ergodicity breaking at low energies when we turn on disorder. We speculate this may be because of proximity to an integrable point (the clean continuum Schwinger model). Level crossings in the clean model support this hypothesis.

Level crossings





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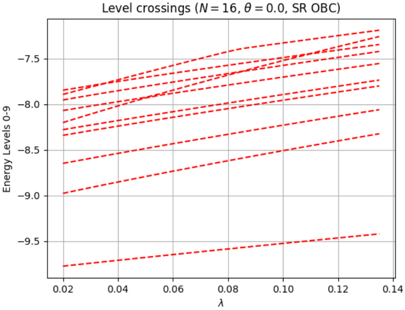

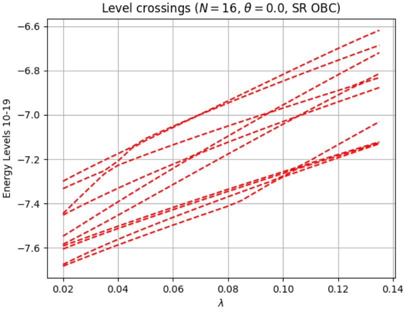

Since $\xi \approx 20$ according to [1], we must probe larger system sizes.

Density imbalance (CR)

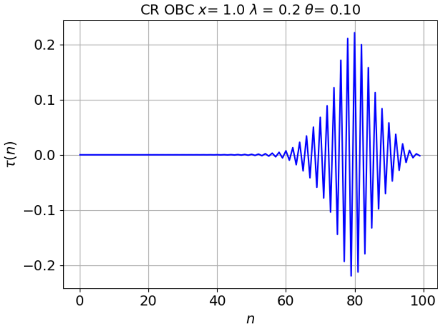

Figure: Localization length shrinks as disorder strength increases. The elementary excitations are charge neutral (dipoles). Parameter regime: $x >> \theta \sim \lambda$.

$$\tau(n) = \langle\psi_1| \sigma_n^z |\psi_1\rangle - \langle\psi_0| \sigma_n^z |\psi_0\rangle \tag{15}$$

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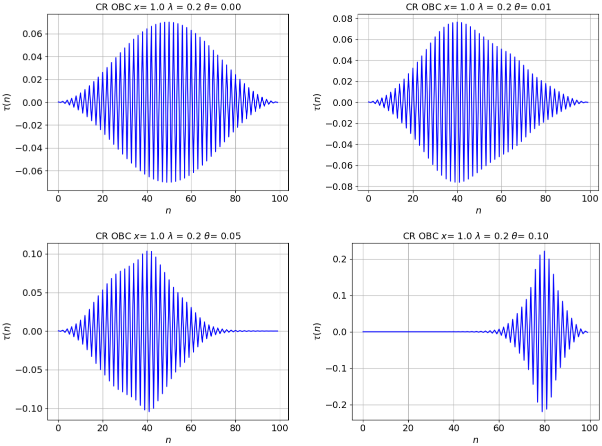
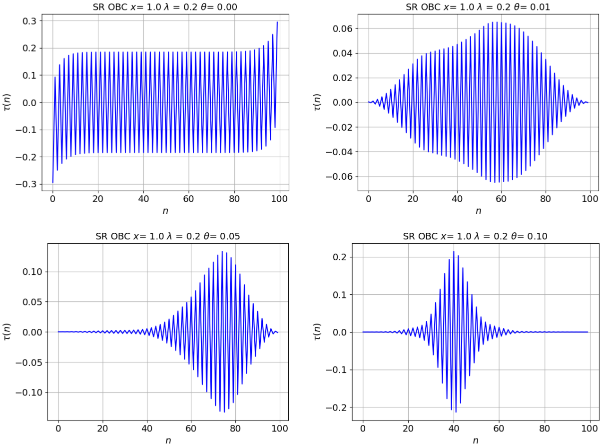
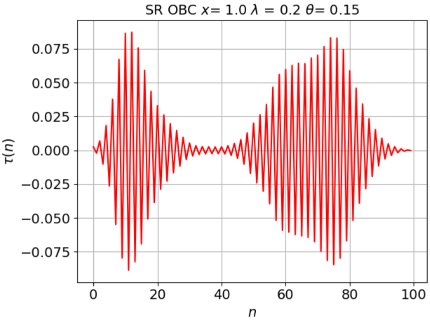
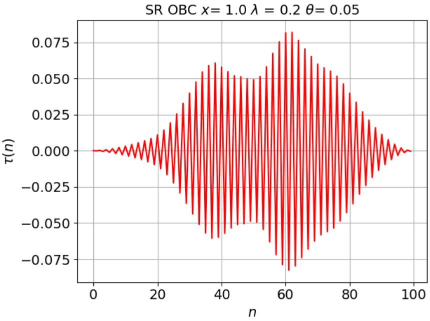


Density imbalance (SR)



Bilocalized states





Localization length scaling

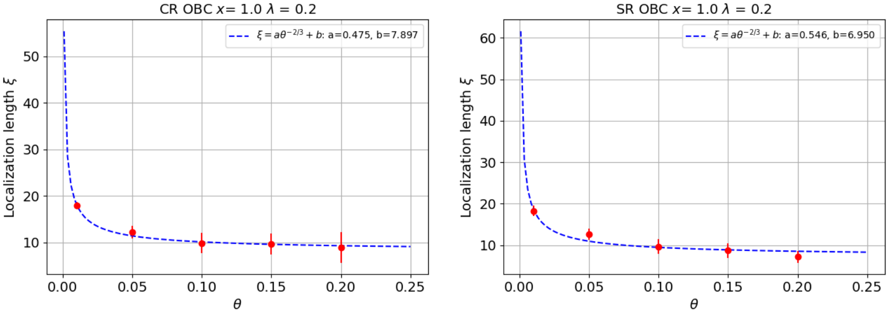


Figure: Comparisons with the predicted localization length of [1].

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  What happens in other parameter regimes?
  What happens at higher energies?

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